Fermi National Accelerator Laboratory

受入
90-12-135
高エネ研図書室

FN-548

Critical Beam-Beam Resonances in the Tevatron

S. Peggs and S. Saritepe
Fermi National Accelerator Laboratory
P.O. Box 500
Batavia, Illinois 60510

August 29, 1990

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

Critical Beam-Beam Resonances in the Tevatron

S. Peggs and S. Saritepe
Fermi National Accelerator Laboratory *
Batavia, Illinois, USA

August 29, 1990

1 Introduction

During the 88-89 Tevatron collider run, 6 antiproton bunches were colliding head-on with 6 proton bunches at 12 crossing points symetrically distributed around the ring. Typical intensities were 7×10^{10} and 2.5×10^{10} particles per bunch for protons and antiprotons respectively. The normalized transverse proton emittance (95 % definition) was typically 25 π mm-mr on both planes and the antiproton transverse emittance was typically 18 π mm-mr. The proton emittance was increased by artificial means to place the antiproton beam in the linear region of the beam-beam force. This procedure, however, yielded a lower initial luminosity. When the emittances were approximately the same the antiproton lifetime was shorter than the proton lifetime. This meant that antiprotons sampling the nonlinear part of the beam-beam force were influenced by resonances. By blowing up the proton emittance the antiproton lifetime was improved and a higher integrated luminosity was achieved. The working point (unshifted horizontal and vertical tunes) was near 19.41.

Operationally, when the proton beam intensity was above 10×10^{10} particles per bunch the average antiproton tune would be near 19.428 (7^{th} order resonance) and its beam lifetime would decrease, again, resulting in a loss of luminosity. This showed that odd-order resonances were not completely suppressed. Since exactly head-on beam-beam interactions at dispersion free regions do not drive odd-order resonances, we are led to believe that one or other of these conditions is not met. The 2/5 (5^{th} order) resonance is even more destructive (i.e., has a larger resonance width). In future collider runs, the orbits of protons and antiprotons will be separated everywhere except at interaction regions B0 and D0. Although separated, proton and antiproton bunches will interact electromagnetically (long-range beam-beam interaction as

*Operated by Universities Research Association,Inc., under contract with the United States Department of Energy

opposed to head-on interaction) at the crossing points. A long-range interaction introduces a dipole field thus causing a closed orbit distortion, and also driving odd-order resonances.

The working point in future collider runs will be near 20.58, again between 7^{th} and 5^{th} order resonances. Due to the helical separation scheme (horizontal and vertical separations oscillating 90° out of phase) the long range beam-beam interactions will not contribute much to the tune shift nor to the tune spread but they will broaden the odd-order resonances.

The distance between the 5^{th} and the 7^{th} order resonances is approximately 0.0286. This gap is not the largest resonance free area (free of resonances lower than 12^{th}) in tune space. There is a larger area near the integer. However, closed orbit and focusing perturbations are very strong there. In principle, it is possible to place the working point near the integer by taking advantage of special orbit and tune correction circuits. The practicality of this approach has been demonstrated in the Tevatron[1]. Although this is a real possibility, in this note we assume that the working point for future collider runs will be between 5^{th} and 7^{th}.

Given the working point and typical tune spreads of about 0.024, it is unavoidable that the 12^{th} order resonance will affect some of the antiprotons. This resonance was certainly felt by the $a > 2\sigma$ particles in the 88-89 run when proton and antiproton emittances were equal. The situation improved when the proton emittance was increased. The question of whether or not the 12^{th} will be harmless in future collider runs remains, since the beam-beam tune shift parameter will be much higher than it was in the 88-89 Run.

In this note, we answer this question by applying the analytic theory of tune-modulated beam-beam resonances[2],[3] to the 88-89 run, to the main injector upgrade scenario and to the speculative scenarios outlined in Reference [4]. The relevant parameters are listed in Table 1. Analytic theory gives a threshold equation (for a given transverse amplitude) by which we can calculate the highest order betatron resonance that allows side-band resonance overlap in the presence of tune-modulation. From here on, we shall call these "critical beam-beam resonances". The analytic theory neglects long-range tune shifts and resonances.

2 Threshold Condition

For a complete discussion of the theory we refer the reader to References [2] and [3]. Here we repeat the threshold condition for synchrobetatron sideband overlap,

$$\left(|\sum^{m} \vec{\xi}_{Ni} | \cdot \sum^{m} | \vec{\xi}_{Ni} | \right)^{1/2} \quad > \quad \frac{1}{4} (\pi q)^{1/4} (Q_s)^{3/4} \left[\frac{\alpha}{N^{3/2} V_N(\alpha) D'(\alpha)} \right]^{1/2} \tag{1}$$

where N is the order and $\alpha \equiv a/\sigma$ is the normalized amplitude of the resonance, and m is the number of head-on beam-beam interactions. It is supposed that, owing to an external modulating source, the perturbed betatron tune is given by

$$Q = Q_0 + q \ sin(2\pi Q_s t) \tag{2}$$

	88-89 Run	Main Injector	$p\bar{p}$ Ultimate	pp Ultimate	
N_p	7.0×10^{10}	3.3×10^{11}	4.0×10^{11}	1.6×10^{11}	
$N_{\bar{p}}$	2.9×10^{10}	3.7×10^{10}	1.1×10^{11}		
Number of bunches per beam	6	36	108	1000	
Total Antiprotons	1.7×10^{11}	1.3×10^{12}	1.2×10^{13}		
ε_p	25	30	30	24	π mm-mr
$\varepsilon_{\bar{p}}$	18	22	30		π mm-mr
β^*	55	50	25	25	cm
Initial Luminosity	1.6×10^{30}	5.7×10^{31}	1.0×10^{33}	1.2×10^{34}	$cm^{-2}s^{-1}$.
$\xi(\bar{p})$	0.002	0.008	0.010	-	
$\xi(p)$	0.001	0.001	0.003	0.005	
$\Delta Q(\bar{p})$	0.024	0.016	0.020	-	
$\Delta Q(p)$	0.012	0.002	0.006	0.010	
Bunch Spacing	3500	395	132	19	nsec
m	12	2	2	2	
Highest order Critical resonance for antiprotons ($\alpha = 3$)	13	13	13	-	
Highest order Critical resonance for protons ($\alpha = 3$)	11	8	10	11	

Table 1: Beam parameters

where Q_0 is the unperturbed betatron tune, q is the amplitude of the modulation (modulation depth), Q_s is the modulation tune, and t is the turn number. The resonance analysis is done at a particular point in the ring and "time" for the purposes of this analysis is discretized. One source of tune modulation is ripple in the current supplied to some of the guide field magnets. A more systematic source is the chromatic tune variation due to energy oscillations (synchrotron oscillations). As the momentum of the particle changes the effective focusing strength also changes, resulting in tune modulation. The detuning function $D(\alpha)$ and the resonance width function $V_N(\alpha)$ are explained below. The $\vec{\xi}_{Ni}$ are the so called resonance vectors

$$\vec{\xi}_{Ni} = \xi_i exp(jN\phi_i) \tag{3}$$

where ξ_i and ϕ_i are the tune shift parameter and the betatron phase of the i'th collision, and $j \equiv \sqrt{-1}$. Properly speaking, $\vec{\xi}_{Ni}$ is a phasor since time (number of turns) is eliminated from this expression.

2.1 Detuning Function and Resonance Width

We restrict our attention to one transverse dimension. Moderate amplitude nonresonant oscillations in a second dimension appear to have little influence on chaotic behaviour in the first dimension[3].

Consider a collider with a single beam-beam collision per turn. The betatron tune of a test particle depends on its amplitude, according to

$$Q(\alpha) = Q_0 + \xi D(\alpha) \tag{4}$$

where $D(\alpha)$ is the so called "detuning function" and ξ is the "beam-beam tune shift parameter" which is equal to the tune shift experienced by a small amplitude particle. If colliding bunches have Gaussian transverse charge distributions of the same size (round beams), the detuning function has the exact analytic form

$$D(\alpha) = 4\alpha^{-2}\left[1 - exp(-\alpha^2/4)I_0(\alpha^2/4)\right] \tag{5}$$

Here I_0 is a modified Bessel function. A beam-beam resonance of order N is present if the tune is equal to a rational fraction m/N at some amplitude α_N. The resonance islands have a full width given by

$$\Delta\alpha = 4\left[\frac{V_N(\alpha)}{\alpha D'(\alpha)}\right]^{1/2} \tag{6}$$

For round beams the "resonance width function" $V_N(\alpha)$ is (even order only)

$$V_N(\alpha) = \int_0^\alpha \frac{8}{\alpha}\, exp(-\alpha^2/4)\ I_{N/2}(\alpha^2/4)d\alpha \tag{7}$$

The detuning function is shown in Figure 1 and resonance island half widths are shown in Figure 2.

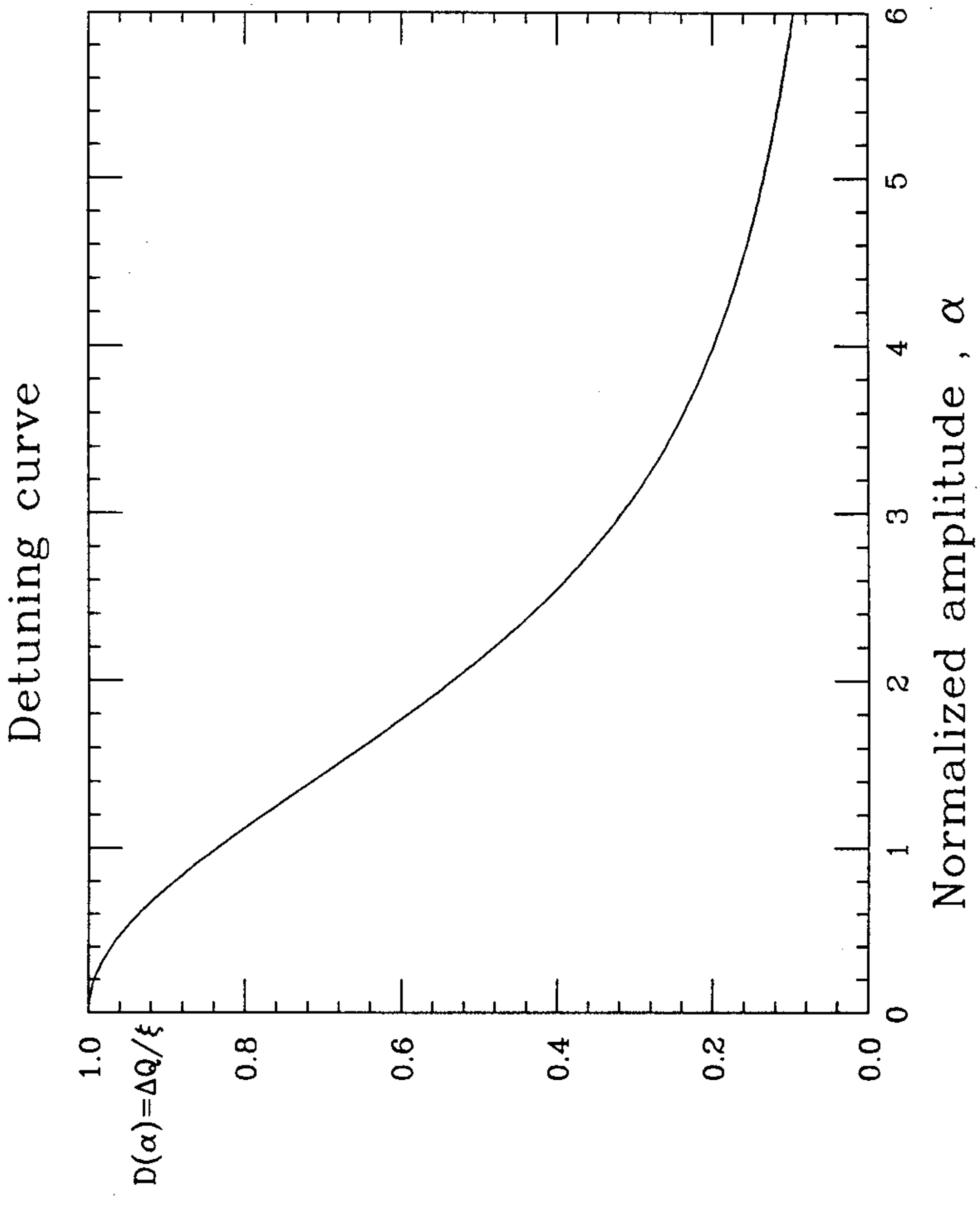


Figure 1: Detuning function.

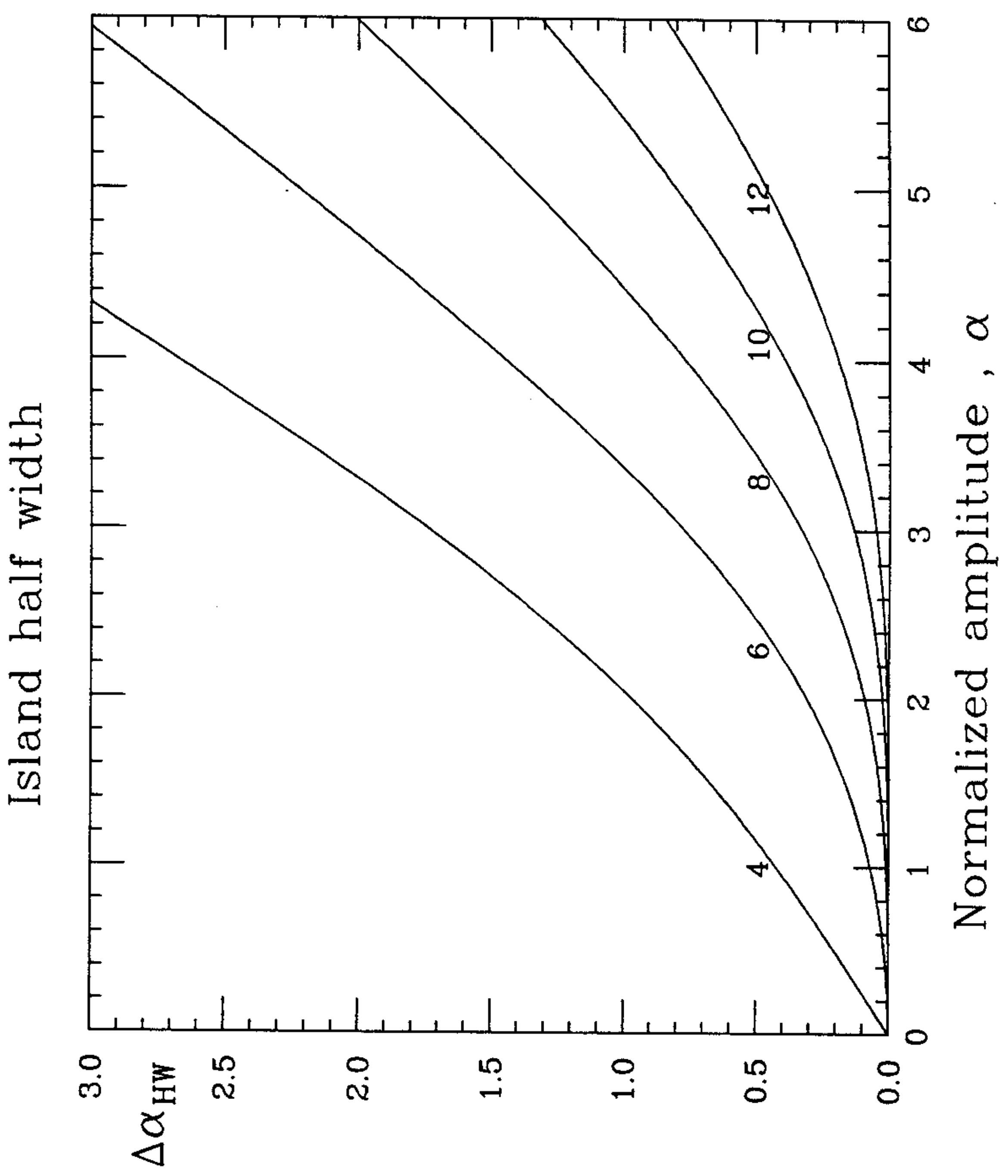


Figure 2: Resonant island half-widths

2.2 Tune Modulation

Tune modulation causes a family of synchrobetatron sideband resonances to appear, at time-averaged tunes of

$$Q(\alpha) = m/N + p\, Q_s/N \tag{8}$$

where p is an integer. This situation is depicted in Figure 3(a,b) where the sideband islands surround the betatron islands. The full width of the p^{th} sideband is given (if the sidebands do not overlap) by

$$\Delta\alpha_{wp} = 4\left[\frac{V_N(\alpha_p)J_p(Nq/Q_s)}{\alpha_p D'(\alpha)\mid_{\alpha=\alpha_p}}\right]^{1/2} \tag{9}$$

Here J_p is the p^{th} integer order Bessel function, and α_p is the betatron amplitude corresponding to this sideband. The magnitude of J_p is of the order of

$$J_p(Nq/Q_s) \approx (Q_s/\pi Nq)^{1/2} \tag{10}$$

if

$$\frac{m}{N} - q < Q(\alpha_p) \;<\; \frac{m}{N} + q \tag{11}$$

and very small if condition 11 is violated. The physical interpretation of this condition is as follows. Because of the tune modulation, the "instantaneous" tune varies between $Q(\alpha) - q$ and $Q(\alpha) + q$. For the resonance to have effect, this tune must cross m/N. So, if $Q(\alpha) < (m/N - q)$ or $Q(\alpha) > (m/N + q)$, the tune never reaches the resonance condition and the sidebands are suppressed. Sidebands are separated in amplitude from each other by

$$\Delta\alpha_s \equiv \frac{\Delta Q}{dQ/d\alpha} = \frac{Q_s}{N\xi D'(\alpha)} \tag{12}$$

As the beam-beam tune shift parameter ξ is increased, the sidebands remain constant in size while their separations decrease. When $\Delta\alpha_s \;<\; \Delta\alpha_{wp}$, the sidebands overlap and a chaotic layer is formed in phase-space flow as shown in Figure 3(d). In other words, there is overlap if

$$\xi \;> \xi_{max} \equiv \frac{1}{4}(\pi q)^{1/4}(Q_s)^{3/4}\left(\frac{\alpha}{N^{3/2}V_N(\alpha)D'(\alpha)}\right)^{1/2} \tag{13}$$

which is very similar to Equation 1 but needs to be generalized to multiple collisions.

The generalized $\Delta\alpha_s$ and $\Delta\alpha_{wp}$ are

$$\Delta\alpha_s = \frac{Q_s}{N\cdot \sum^{m} \mid \vec{\xi}_{Ni} \mid \cdot D'(\alpha)} \tag{14}$$

$$\Delta\alpha_{wp} = 4\left[\frac{\mid\sum^{m}\vec{\xi}_{Ni}\mid \cdot V_N(\alpha)J_p(Nq/Q_s)}{\sum^{m}\mid \vec{\xi}_{Ni}\mid \alpha D'(\alpha)}\right]^{1/2} \tag{15}$$

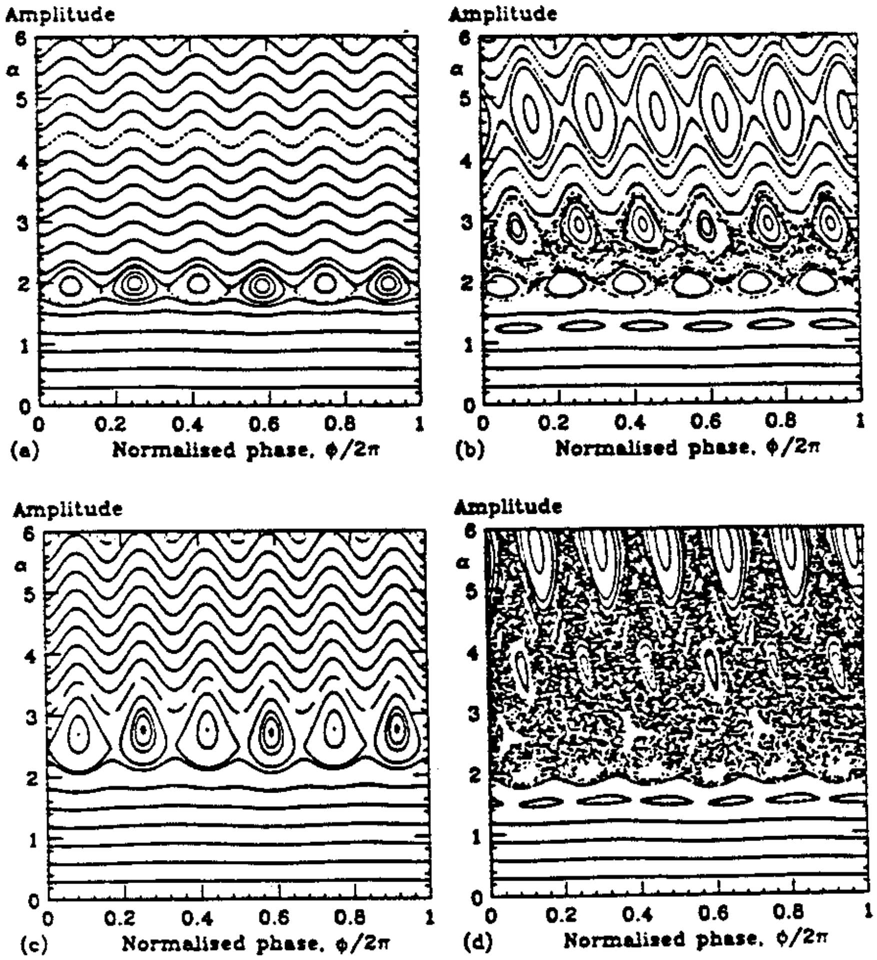


Figure 1: Simulated trajectories tracked for 2000 modulation periods, with $Q_s = 0.005$ and an unshifted tune of 0.331, near a sixth-order beam-beam resonance. The two left figures have no tune modulation, while the two right figures have modulation amplitude $q = 0.001$. The two top figures have a tune shift parameter of $\xi = 0.0042$, while the two bottom figures have a value $\xi = 0.0060$. Side bands $p = +1, 0, -1$, and -2, visible in (b) at increasing amplitudes, overlap and are submerged in a chaotic sea in (d).

Using the overlap condition $\Delta\alpha_s < \Delta\alpha_{wp}$ and Equations 14,15 and rearranging we obtain Equation 1, the threshold equation. Given the order of the betatron resonance N, the particle amplitude α, the tune modulation frequency Q_s and depth q, the threshold equation tells whether the beam-beam strength parameter ξ is large enough to cause an overlap of sideband resonances.

2.3 Summing the Beam-Beam Resonance Vectors

We are now ready to calculate the terms on the left hand side of Equation 1. The calculation of $|\sum^m \vec{\xi}_{Ni}|$ requires the knowledge of phases at crossing points. There is typically a 10% measurement error of the lattice functions, and it is difficult to know the phases exactly enough at the crossing points. We simply take the root mean square average of the resonance vectors $\vec{\xi}_{Ni}$, namely, we approximate

$$|\sum^m \vec{\xi}_{Ni}| \approx (m)^{1/2}\xi \tag{16}$$

and the other summation is easier since the phase information is not needed.

$$\sum^m |\vec{\xi}_{Ni}| = m\xi \tag{17}$$

In all the upgrades, the average long-range separation will be about 5σ, therefore the contribution to the total tune shift will be very small[6].

$$\sum^m |\vec{\xi}_{Ni}|_{\text{long-range}} \approx 0 \tag{18}$$

Also, due to the helical separation arrangement the resonance vectors add up approximately to zero for the long-range collisions.

$$|\sum^m \vec{\xi}_{Ni}|_{\text{long-range}} \approx 0 \tag{19}$$

3 Critical Resonances

Critical resonances are calculated graphically from Figure 4 where the right hand side of Equation 1 is plotted for different N. Although curves for odd-order resonances are not shown in Figure 4 we assume, extremely conservatively, that they are placed in between the even-order resonances having the same form. If the threshold condition lies half way between orders 10 and 12, for example, we report it as 11, even though the long-range interactions are too weak to drive odd-resonances. Long-range interactions, however, are not the only sources of odd-resonances, apart from field errors, there are sources arising from beam dynamics. Empirical evidence from CERN show that, protons and antiprotons can have closed orbits different by as much as 0.1σ, due to their differing tunes, even when separation is off. This means that beam-beam interactions at detector locations are not really head-on as they are supposed to be. Calculations[5] show that an effective separation of 0.1σ, causes further closed orbit distortions (typically 0.2σ) and drives odd-resonances. Although in practice, orbit distortions at B0 and D0 will be corrected, one simply cannot ignore odd-resonances altogether. So, to summarize, we ignore long-range interactions in the summation of beam-beam resonance vectors (left hand side of the threshold equation) but do not ignore odd-resonances (right hand side of the threshold equation) since they can be driven by other sources.

The curves in Figure 4 have been calculated using realistic Tevatron parameters. For instance, a chromaticity of $\Delta Q/(\Delta p/p) = 5$ and $\sigma p/p = 1.5 \times 10^{-4}$ was used, assuming that the source of the tune modulation was synchrotron oscillations at a frequency of 37 Hz in the Tevatron at 900 GeV. These numbers translate into $Q_s = 0.00075$ and $q = 0.00075$. Another set of curves are displayed in Figure 5 with $Q_s = 0.00075$ and $q = 0.00150$ where the chromaticity was 10 units.

4 Bed-of-Nails Plots

Critical resonances are displayed in Figures 6, 7 and 8. In these figures, nails have different heights representing the order of the resonance. Even-order resonances are shown by solid lines and odd-order resonances by dashed ones. Lower order resonances are represented by taller nails. Loosely speaking, the height of a nail corresponds to a resonance width, although, we simply drew the heights in proportion to the inverse of the resonance order N. The tune spread is shown by the horizontal error bar, the position of which carries crucial information. Its vertical position indicates the critical resonance for $\alpha = 2.5$. The vertical error bar shows the range of critical resonances for particles in the range $\alpha = 2$ to $\alpha = 3$. The point where the horizontal and the vertical error bars cross each other, roughly gives the beam-beam shifted average tune. The working point (unshifted tune) is near the left edge of the horizontal bar.

Figure 6 depicts the situation in the 88-89 collider run. The analytic theory of tune modulated beam-beam resonances correctly predicts the lack of importance of the 12^{th} order resonance, since it only just affects $\alpha = 2.6 - 3.0$ antiprotons. For $\alpha = 2$ antiprotons the theory predicts no trouble from 12^{th}, which was the case when proton emittance was increased artificially, effectively making all antiproton amplitudes $\alpha \equiv a_{\bar{p}}/\sigma_p < 2$. Protons in the 88-89 run were comfortably away from the 12^{th} since the antiproton intensity was low and the beam-beam tune shift per crossing experienced by protons was small.

5 Conclusions

Examining the remaining "bed-of-nails" plots we find that for the collider runs with the Main Injector, the horizontal error bar (tune spread) is smaller and the vertical error bar (range of critical resonances for amplitudes $\alpha = 2 - 3$) is approximately the same compared to those of the 88-89 run. Having a smaller tune spread is certainly an improvement since we gain freedom to adjust the working point. The size of the vertical error bar being equal to that of the 88-89 run is also good news since it means that the 12^{th} order resonance will only affect the antiprotons in the transverse tails. The bad news is that these antiprotons in the tails are not meant to be influenced by the 12^{th} since the proton emittance is already made larger with the anticipation of stronger beam-beam resonances (The nominal parameters for this collider run are: normalized transverse proton emittance= 30 π mm-mr, antiproton emittance= 22 π mm-mr, proton/bunch= 33×10^{10}, and chromaticity=5 units). Therefore, it may be necessary to increase the proton emittance further by artificial means to effectively reduce the antiproton amplitudes. This solution is not very desirable, however, since it reduces the luminosity.

A slight improvement may also be made by increasing the chromaticity. This conclusion is reached from an examination of Figures 4,5 that show how curves shift to the right when chromaticity is increased from 5 units to 10 units. Here we reiterate that it is easier to have sideband overlap with resonances of order less than the order quoted as "critical". So, by lowering the order of the "critical" resonance we are lowering the probability of sideband overlap. This corresponds to a vertical upward shift of the "cross" in "bed-of-nails" plots. The cost of the "chromaticity cure" is a larger tune spread, in other words, the horizontal error bar gets bigger. This cure could not be applied in the 88-89 run because the tune-spread was as large as the available tune space of about 0.024. In the collider run with the Main Injector, the tune spread will be 0.016 (assuming the nominal parameters) so there is room for additional spread from chromaticity. The effect of the chromaticity on the feed-down correction circuit is of second-order, hence negligible[7] so the "chromaticity cure" might be useful in future collider runs.

Selection of the working point in the future collider runs is also very important since we have some freedom in moving the "cross" in "bed-of-nails" plots horizontally. By carefully adjusting the the working point one can avoid the 12^{th} order resonance as suggested by the horizontal placement of the "cross" in Figure 7. The working point will be studied in simulations[8] first, and will be fine tuned during the collider operations.

The critical-resonance picture for the speculative upgrade scenarios is shown in Figure 8. The "$p\bar{p}$ Ultimate" scenario suffers from the strong presence of the 12^{th}. In the "pp Ultimate" scenario beam-beam resonances will be very weak.

References

[1] R.Johnson and P.Zhang; "A new Tevatron Collider Working Point near the Integer"; Proceedings of the 1989 IEEE Particle Acc.Conf.(Chicago), p:806.

[2] S.Peggs and R.Talman; "Nonlinear Problems in Accelerator Physics"; Ann.Rev.Nucl.Sci., 36,p:287-325 (1986).

[3] S.Peggs; "Hadron Collider Behaviour in the Nonlinear Numerical Model EVOL"; Particle Accelerators, 17, p:11-50 (1985)

[4] G.Dugan, S.Holmes and S.Peggs; "Ultimate Luminosity in the Tevatron Collider"; Proceedings of the 1990 Snowmass Workshop.

[5] L.Michelotti and S.Saritepe; "Exploratory Orbit Analysis of Tevatron Helical Upgrade;One:A First Look"; Fermilab Report TM-1603. L.Michelotti and S.Saritepe; "Orbital Dynamics in the Tevatron Double Helix"; Proceedings of the 1989 IEEE Part.Acc.Conf.(Chicago) p:1391-1393 (F.Bennet and J.Kopta eds).

[6] S.Saritepe, L.Michelotti and S.Peggs; "Long-Range Beam-Beam Interactions in the Tevatron: Comparing Simulation to Tune Shift Data"; Proceedings of the 1990 Eurepean Part.Acc.Conf.(Nice,France).

[7] G.Goderre, private communication.

[8] S.Saritepe; "Beam-Beam Simulation Results for the Future Collider Runs in the Tevatron"; To be published.

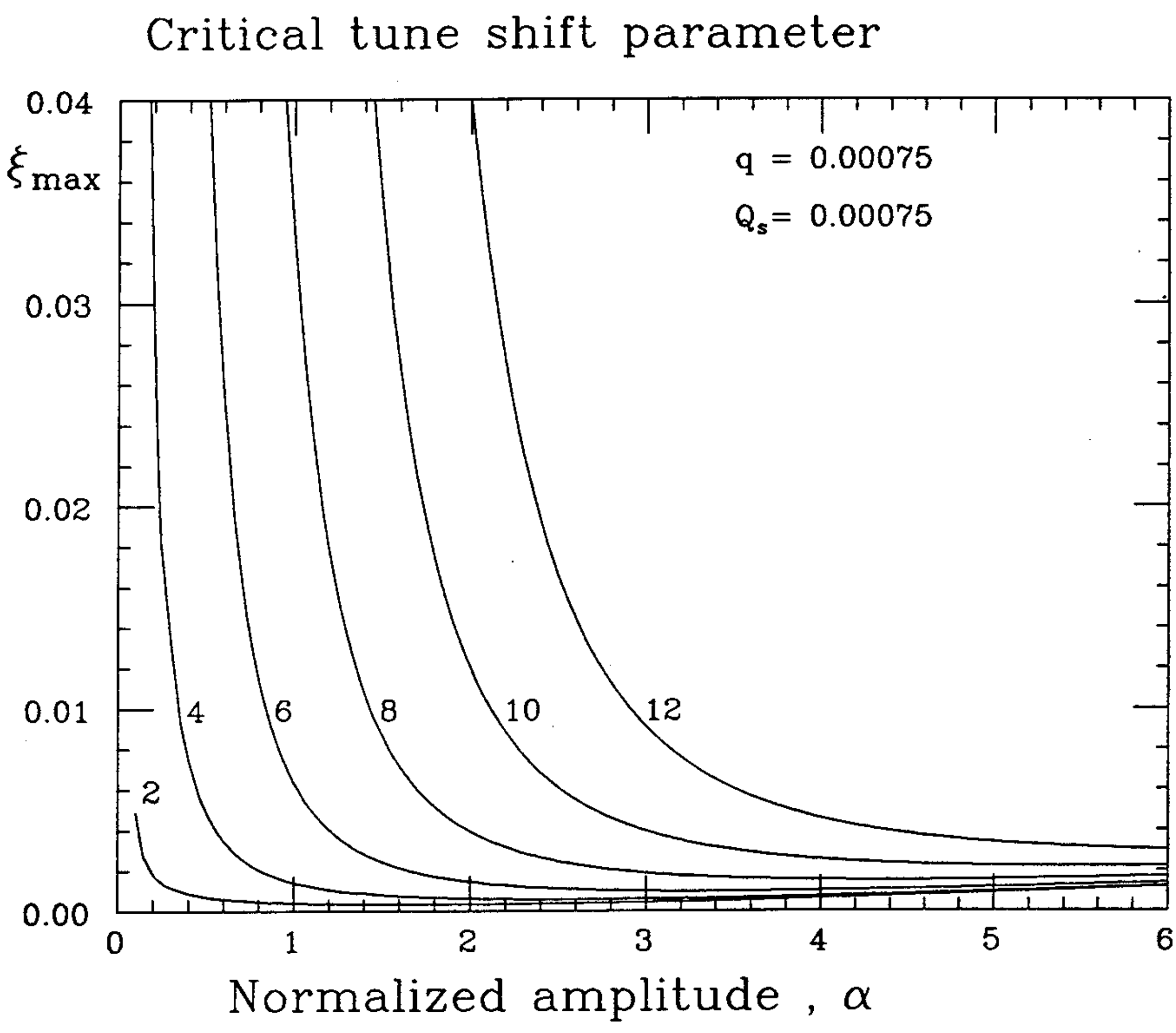


Figure 4: Right hand side of Equation 1 plotted for various N . Chromaticity = 5 units.

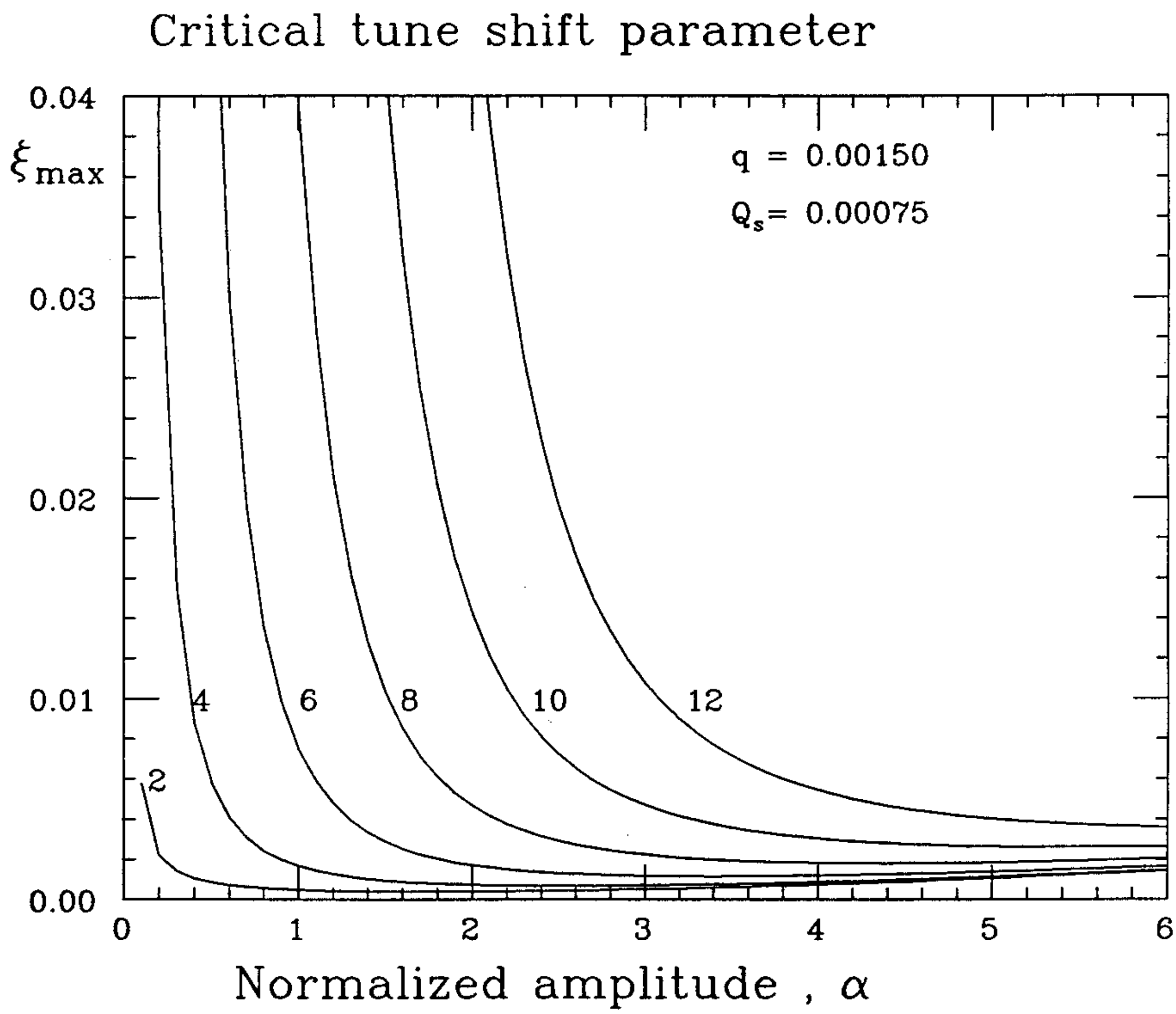


Figure 5: Right hand side of Equation 1 plotted for various N . Chromaticity = 10 units.

88-89 Run

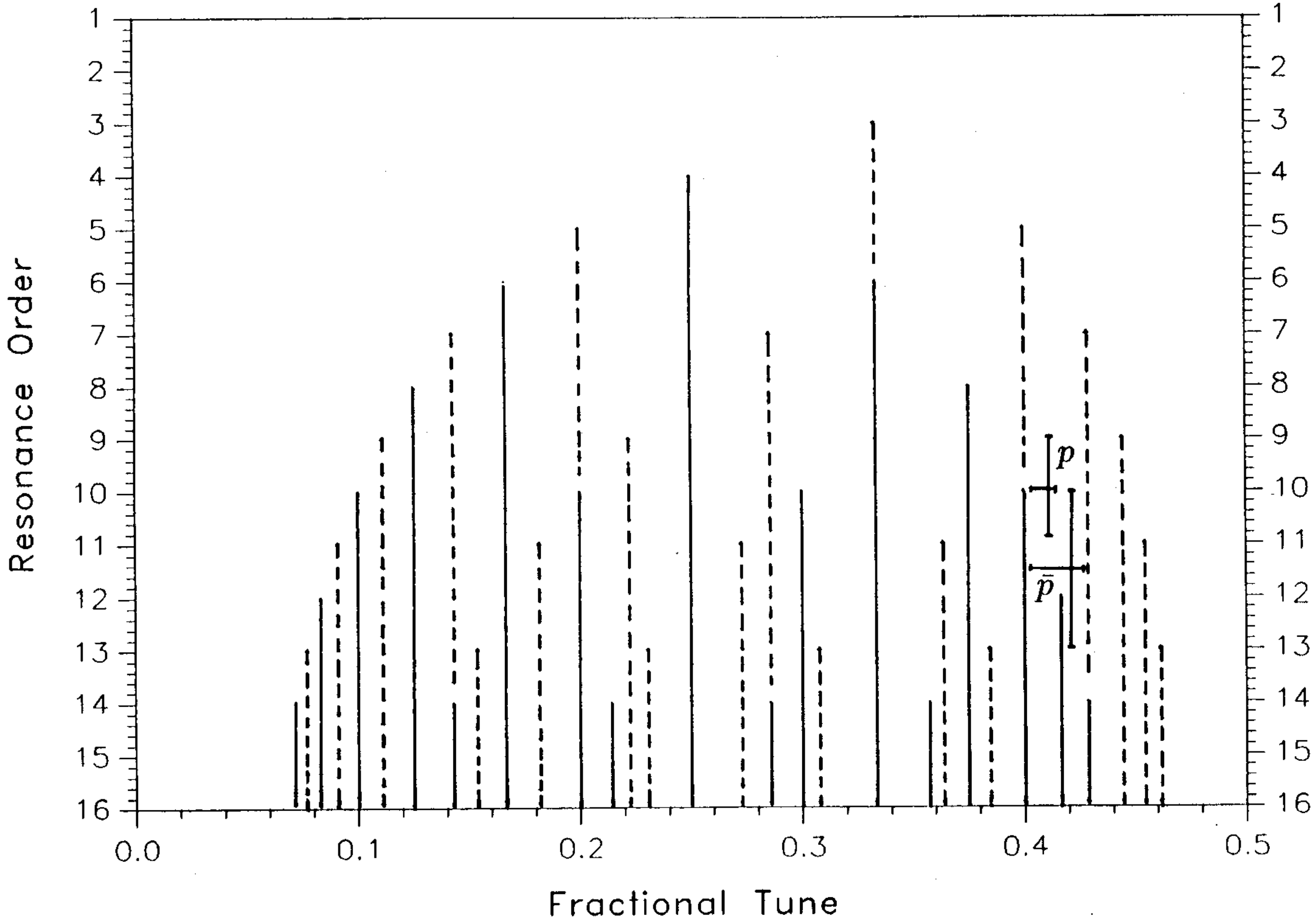


Figure 6. Bed-of-Nails plot for the 88-89 Collider Run.

Main Injector

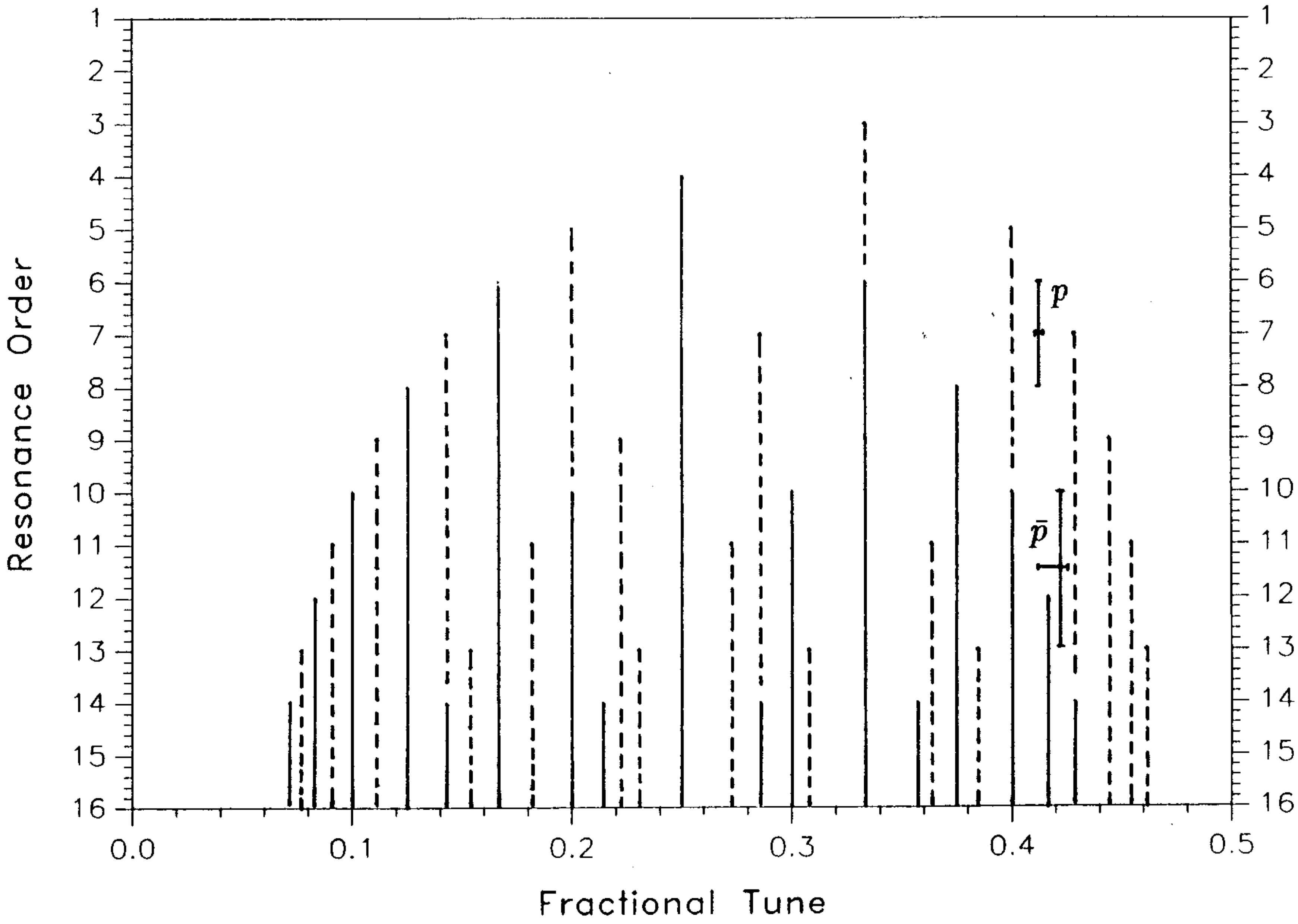
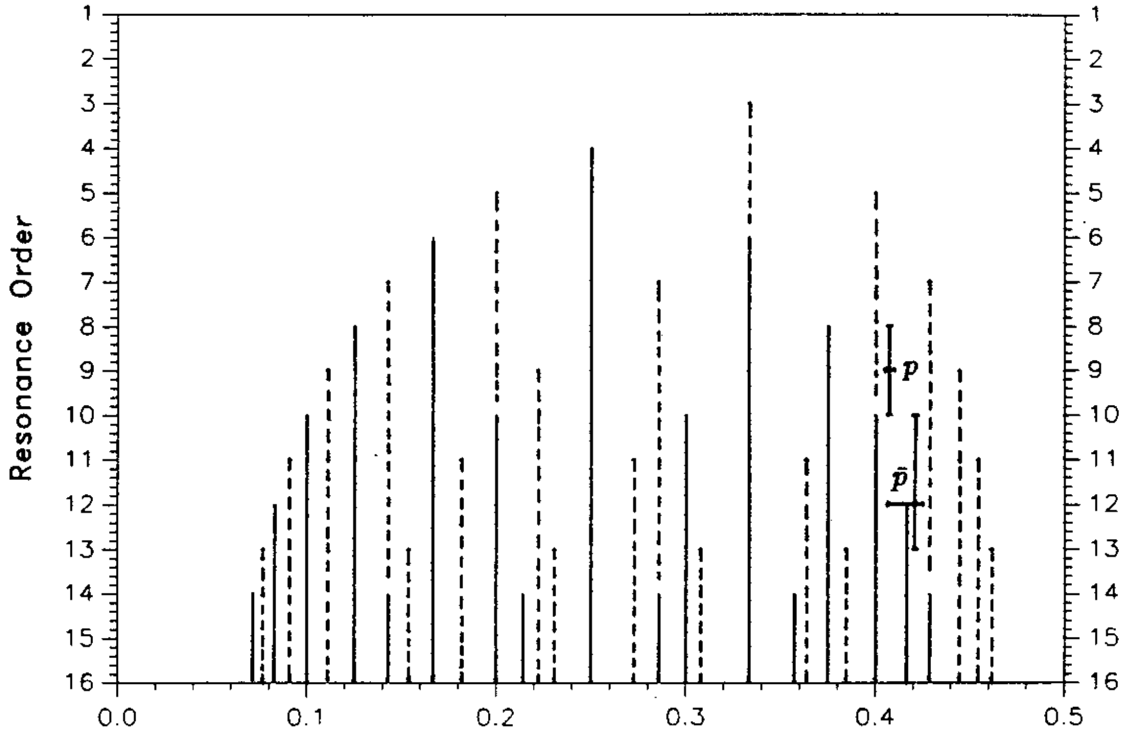


Figure 7. Bed-of-Nails plot for the Collider Run with Main Injector.

$p\bar{p}$ Ultimate



pp Ultimate

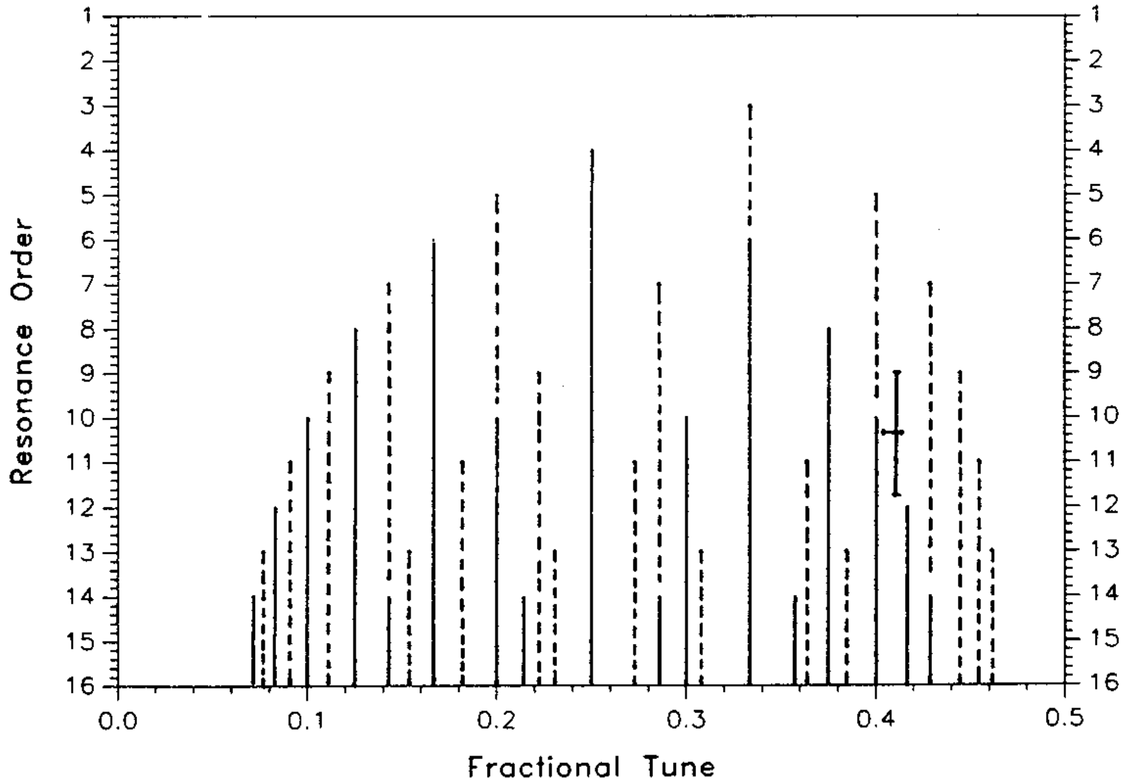


Figure 8: "Bed-of-Nails" plots for the speculative upgrade scenarios.